MATHS CHALLENGE SOLUTIONS – PART 1

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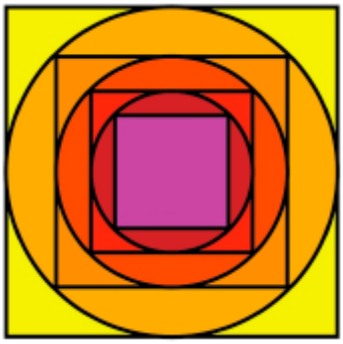
Below are our detailed solutions to The Age Maths Challenge (24 November 2008). As we remarked at the time of the Challenge, very few of the 60 puzzles are our own creations. Most are based on common mathematical principles and/or are versions of well-known puzzles. Some have appeared in the collections of the great puzzlemasters Martin Gardner and H. E. Dudeney and Sam Loyd. Even for those puzzles, the original source is usually not evident.

1. If the purple square has area 1, what is the area of the outer yellow square?



Answer: The large square has area 8. This is straight-forward to calculate, but it is also possible to see at a glance.

The diagrams show that the outer square has exactly twice the area of the inner square, 8 triangles instead of 4.

Since we have four nested squares, we simply have to double three times to arrive at our answer $1 \times 2 \times 2 \times 2 = 8$.

2. In *The World is Not Enough,* a bomb is traveling at 70 miles per hour, and is 106 miles from its target. James Bond immediately declares that they have 78 minutes to stop it. Is he correct?

Answer: Distance divided by speed gives a time of 106 m / 70 m/h = 1.514 hours. This is about 91 minutes. So, James is wrong, but at least he underestimated the time to stop the bomb.

3. How can you make sense of this strange sum?

$$\begin{array}{r} 340 \\ 437 \\ +\quad 78813 \\ \hline 837378178 \end{array}$$

Answer: Looking at the sum in a mirror, all the digits turn into letters. You'll discover that indeed

0NE+TEN+EIGHT=NINETEEN (in letters).

Playful writings such as this, based upon real or forced symmetries, are called ambigrams. See
http://en.wikipedia.org/wiki/Ambigram and
http://www.qedcat.com/books.html#eye.

4. A die fits exactly into its cubical box. In how many different ways can you place the die in the box?

Answer: Imagine one of the sides of the cubical box to be open. There are 6 faces which might be showing. Whatever face is showing, there are 4 possible rotations of that face. This gives a total of $4 \times 6 = 24$ ways to fit the die into the box.

5. Imagine a loop that fits snugly around the Equator of the Earth. Now lengthen this loop by one meter and imagine the new loop hovering above the ground. How far is the loop above the ground? What if you do the same with a rope around a football?

Answer: Circles are easy. Since the circumference of a circle of radius R is $2\pi R$, adding 1 meter to the radius increases the circumference by 2π meters. And, lengthening our rope by 1 meter, the same equation shows the radius will increase by $1/2\pi$ meters, about 16 centimeters. In particular, this conclusion does not depend upon the original size of the circle. See
http://www.qedcat.com/mathsnacks/pi.html.

The details are harder, but in fact the very same calculation works for any convex figure: the loop lengthened by 1 meter will again hover about 16 centimeters above the footy.

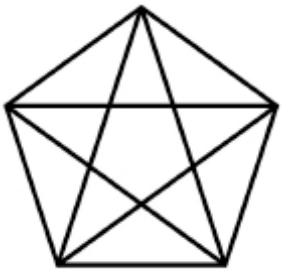
6. Start with the number 2: then adding 2 gives 4, and also multiplying by 2 gives 4. Can you do this for any other number? For example, starting with 9 can you find a second number so that either adding or multiplying 9 by this second number gives the same answer?

Answer: Calling the second number N, this problem translates into the equation $9+N=9 \times N$. Solving we arrive at our answer $N = 9/8$. The same calculation works for any number replacing 9, except for 1.

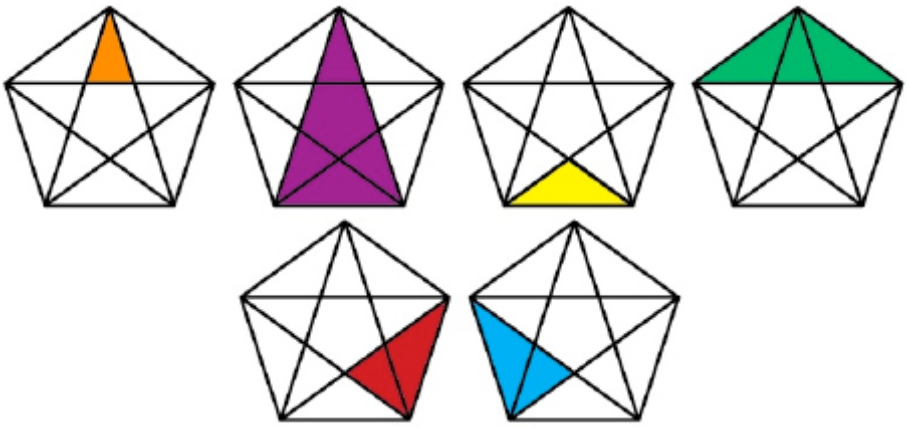
7. Ken buys two cars, and then sells them for $6000 each. One car he has sold for a 25% profit and the other for a 25% loss. Overall, has Ken made a profit or a loss?

Answer: Suppose Ken paid A and B for the two cars. Then the two equations become A (1+1/4)= 6000 and B (1–1/4)= 6000. So, A = $4800 and B = $8000. This means Ken made profit of $1200 on one car and a loss of $2000 on the other, overall a loss of $ 800.

8. How many triangles are in this figure?



Answer: The 7 different types of triangles are indicated below. Rotating the diagrams given a total of 7 × 5 = 35 triangles.



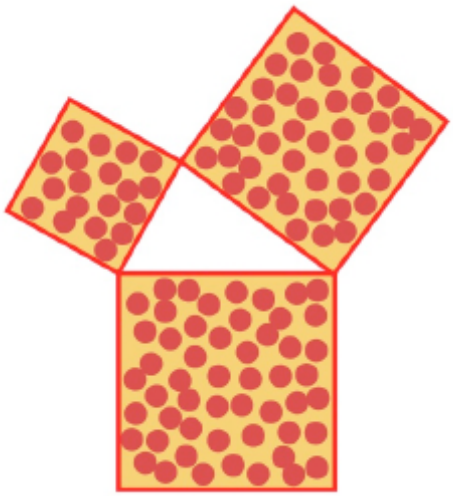
9. In the *Futurama* episode *A Fishful of Dollars*, Phillip Fry collects the compound interest on the 93 cents he left in his bank account 1000 years earlier. His annual interest rate has been 2.25%. The suggested balance is 4.3 billion dollars (all of which he blows on a can of anchovies). How close is this to the true value?

Answer: $\$0.93 \times (1.0225)^{1000} = \$4{,}283{,}508{,}449.71$.

10. In the movie *Il Posto*, Domenico applies for a job, and has to solve the following problem. A roll of copper wire is 520 meters long. Three-quarters of it is cut off. Of the remainder, we cut off four-fifths. How many meters of wire are left on the roll? Domenico is given an hour to solve the problem (and gets it wrong). Can you do it?

Answer: 520 metres × 1/4 × 1/5 = 26 metres. (In the movie Domenico's answer is 24).

11. Peter Thagoras's Pizza Shop sells square pizzas. A large pizza costs the same as a medium and a small pizza together. If your aim is to get value for money, how do you quickly decide whether to buy the large, or the medium and the small?



Answer: Simply fit the pizza trays together into a triangle as shown in the diagram. If the largest angle is a right angle, then Pythagoras's theorem tells you that it does not matter which combo you choose. If the angle is greater than 90 degrees you go for the large pizza (Think of the right angle as expanding. Then the small and medium pizzas stay the same size while the large gets larger). If the angle is smaller than 90 degrees the small + medium combo is the better deal. Also, in theory, if the large pizza is really large, then you won't be able to make a triangle. In this unlikely case the large pizza is obviously a terrific choice.

12. It is raining at midnight. Will we have sunny weather in 72 hours?

Answer: Maybe at the North Pole or South Pole, but not otherwise (because after $72 = 3 \times 24$ hours it is midnight again).

13. $1=2+2-2-2/2$, $2=2+2+2-2-2$, $3=2+2-2+2/2$. Can you similarly write 4, 5, 6, 7, 8, and 9 using exactly five 2s, and using only the basic arithmetic operations +, −, × and ÷ ?

Answer: $4 = 2 + 2/2 + 2/2$, $5 = 2 + 2 + 2 - 2/2$, $6 = 2 + 2 + 2 + 2 - 2$, $7 = 2 \times 2 \times 2 - 2/2$, $8 = 2 \times 2 \times 2 + 2 - 2$, $9 = 2 \times 2 \times 2 + 2/2$.

14. If a coin rolls without slipping around another coin of the same size, how many times will it rotate while making one revolution? How many revolutions will the coin make if it rolls around 2 coins of the same size that are placed side by side?

Answer: 2 and 8/3. See http://www.qedcat.com/mathsnacks/solutions.html for the gory details.

15. In the horror movie *Cube*, the victims are trapped in a maze of rooms. The maths wiz of the group concludes that a room is booby-trapped if any of the three-digit numbers labeling the room is prime. Here are a few sets of these room numbers. Assuming the maths wiz is correct, determine (without a calculator!) if these rooms are safe: a) 565, 472, 737; b) 476, 809, 539; and c) 212, 373, 649.

Answer: 565 is a multiple of 5, 472 is even, and $737 = 11 \times 67$. Hence this room is safe. The second and third sets of numbers contain prime numbers (809 and 373) and are therefore deadly.

16. In the movie *Little Big League*, the baseball team is presented with a word problem: If Joe can paint a house in 3 hours and Sam can paint it in 5 hours, how long does it take for them to do it together? The team struggles with it. Can you do it?

Answer: In 15 hours they can together paint 8 houses. This amounts to 1 7/8 hours for each house.

17. What is the sum 1+3+5+… of the first thousand odd numbers?

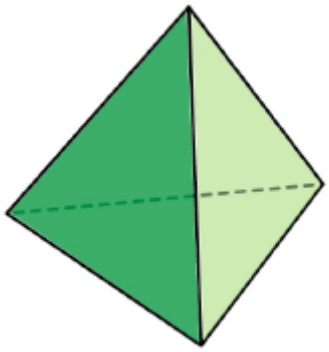
Answer: Note that the numbers of circles in the L-shapes are consecutive odd numbers. Therefore the diagram shows at a glance that the sum of the first N odd numbers is simply N^2 (the number of circles in the square). Since we are interested in the sum of the first 1000 odd numbers the answer to this problem is $1000^2 = 1{,}000{,}000$.

18. In the movie *Rules of Attraction*, Lara is discussing safe sex: "If a condom is 98% safe, and he wears two, then you're 196% safe." Hmm. How safe is it really to wear two condoms?

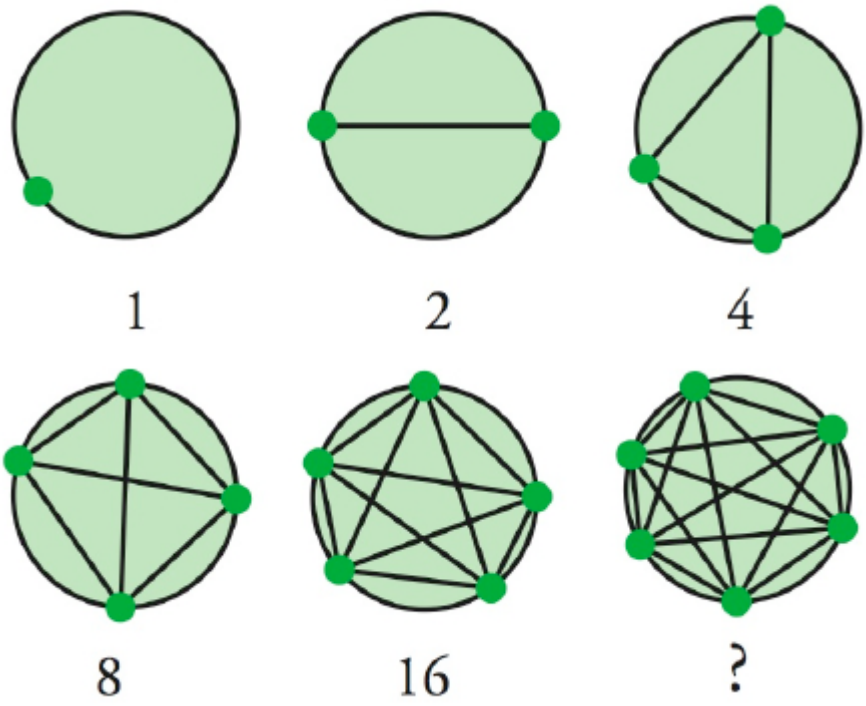
Answer: The probability that one condom is NOT safe is 0.02 (= 2 %). Therefore, the probability that two condoms are not safe is $0.02 \times 0.02 = 0.0004$ (0.04 %). (Well, we're assuming "ideal circumstances"). Consequently, the probability that the two condoms together are safe is 100% – 0.04% = 99.96%.

19. A gardener was instructed to plant four shrubs at equal distances from each other. How did she do it?

Answer: Plant the shrubs at the corners of a tetrahedral mound. A tetrahedron is a four-sided pyramid with all edges of equal length; see the diagram. This means that its four corners are at equal distance from each other. Note that it would be impossible to plant five shrubs at equal distances (unless the garden happened to be 4-dimensional).



20. What's the next number?



Answer: There are 1, 2, 4, 8, 16, and 31 areas in the different diagrams. Therefore the answer to our question is 31. This is the maximum number of regions. Note that if we had spaced the dots equally around the circle then the next number would have been 30. See

http://www.qedcat.com/mathsnacks/perfect_puzzles.html

This puzzle is known as *Moser's Circle Problem.* Its solution, the maximum number of regions if we join *N* dots placed around a circle, is

$$\frac{1}{24}\left(N^4 - 6N^3 + 23N^2 - 18N + 24\right)$$

If the dots are equally spaced and *N* is odd then the number of regions is given by this same formula. For an even number of equally spaced dots there is no simple formula that counts the number of regions.